Study of travelling waves in a neuron-astrocyte model for spreading depression

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Summary:
The use of quantitative tools to tackle relevant problems in biomedicine and in particular, neuroscience, has boosted in the last years and is becoming a growing area of research. This project focuses on the brain waves that occur in migraine and more severely, in ischemia.

The cortical spreading depression and the spreading depolarization correspond to rapid depolarization waves that slowly spread through the cells of the cerebral cortex. Cortical spreading depression typically arises in normal tissue, as in migraine, whereas spreading depolarizations typically arise in metabolically compromised tissue, as in ischemia [1]. Experimental studies have indicated that astrocytes (neuroglia cells) play a fundamental role in preventing the propagation of these waves. Nevertheless, the classical mathematical models of spreading depression do not incorporate the astrocytes, and only very recently some models have been proposed that incorporate them together with neurons, leading to new mathematical questions.

The project proposes to work on a biophysical model of a neuron-astrocyte network developed in [2], consisting of a system of nonlinear diffusion equations (PDEs). The objective is to study the existence, stability, propagation speed and duration of the traveling waves in this model as a function of certain parameters that control the astrocyte dynamics. The identification of the most influential factors in the spreading of the waves could be of special relevance for other fields such as biomedicine.

The mathematical approach will combine tools from the theory of non-smooth systems with methods of singular perturbation theory [3]. The existence of a traveling wave in the original PDE corresponds to a heteroclinic connection to the ODE. These connections are difficult to obtain analytically. In this model, with variables that evolve on different time scales, we will break the analysis in different parts of the heteroclinic trajectory. So, the system can be simplified with simpler functions in each subdomain, which yields a piecewise smooth dynamical system. This strategy will allow us to maintain the original form of the equations, while providing the solution of the travelling wave in the singular perturbation limit. Then, we will develop a mathematical theory that proves the existence of this wave away from the limit. The project will combine analysis with numerical methods, which will complement each other to progress further along the project. We expect that the analytical methods developed for this project will extend the existing results in singular perturbation theory beyond cases that have not been solved yet.

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