ARITHMETIC EQUIDISTRIBUTION

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This project concerns the study of the spatial distribution of the Galois orbits of points in algebraic varieties that are subject to arithmetic conditions. The goal is to explore the distribution of these Galois orbits in situations that go beyond our current knowledge and are poorly understood. The obtained results might have interesting applications to the Diophantine and Arakelov geometry of varieties, and in the arithmetics of dynamical systems.

A cornerstone in the study of the distribution of algebraic integers is a result obtained by Fekete and Szegö in the 1950's, showing that the property that a (symmetric) subset of the complex plane contains an infinite number of Galois orbits of algebraic integers is intimately related to its electrostatic capacity. This result has been widely extended, mainly by the work of Cantor and of Rumely, to arbitrary algebraic curves, adelic subsets and locations of the Galois orbits in the different *v*-adic analytifications of the curve. This are very concrete results whose proof involves tools from potential theory on Riemann surfaces and on Berkovich anaytic spaces.

Several open questions remain open, most notably the extension of the Fekete-Szegö theorem to higher dimensional varieties instead of algebraic curves. One might hope to tackle this problem, at least in the Archimedean setting, using a suitable extension of the logarithmic potential theory. Another important open problem concerns the characterization of the distributions on the complex plane that arise as limits of Galois orbits of algebraic integers.

Another central result in this direction is the Szpiro-Ullmo-Zhang equidistribution theorem and its continuations. This series of result shows that the Galois orbits of points of small *height* (namely, the arithmetic complexity of the point) converge to the Monge-Ampère distribution associated to the considered height function. This is a key tool in arithmetic geometry with applications in the context of the Manin-Mumford conjecture and related ones, including the Lehmer and Bogomolov problem, the Zilber-Pink conjecture. It also has strong implications in the study of (algebraic) dynamical systems, in particular to the distribution of preperiodic points and of families of post-critically finite dynamical systems.

Currently, the application of these equidistribution theorems is limited to the so-called *quasicanonical* heights, that is, height functions that are extremal with respect to Zhang's inequality. Recent results by Burgos Gil, Philippon and Sombra settled the arithmetic equidistribution problem for toric varieties and height functions, showing a variety of interesting behaviors appearing in the non-quasicanonical situation.

Almost nothing is known for varieties and height functions that are not toric. In this situation, understanding the equidistribution properties of Galois orbits of points of small height beyond the quasi-canonical case is a challenging problem where any progress can be significant. A basic step in this direction would be the computation of the essential minimum of a height function and, in particular, its relation with the corresponding arithmetic Okounkov body. We expect that ourproposed results in the direction of the Fekete-Szegö problem might be instrumental to the solution of this problem and related ones.