Fractional diffusions and applications

Group Leader

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Related links to the position (optional)

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Suitable masters degrees from candidates: Mathematics

Area of Knowledge:

- Life Sciences
- Physical Sciences, Mathematics and Engineering

Group of disciplines:

LIFE SCIENCES

Medicine, Public Health, Sport Sciences, Nutrition, Clinical Psychology, Health Management
Animal, Plant, Environmental Biology, Physiology, Ecology and Conservation
Human Biology, Microbiology, Molecular Biology, Genetics, Cellular Biology, Genomics and Proteomics, Biochemistry
Agriculture, Veterinary Science, Animal Production, Forestry
Biotechnology, Bioinformatics, Pharmacy, Food Technology

PHYSICAL SCIENCES, MATHEMATICS AND ENGINEERING

Theoretical and Applied Mathematics, Computer Sciences
Physics
Geology, Earth Sciences, Environmental and Atmosphere Sciences, Mines, Geological Engineering, Oceanography, Hydrology
Civil and Construction Engineering, Energy, Nuclear Energy and Renewable Energy Engineering
Chemistry and Chemical Engineering
Telecommunications, Electronics, Robotics, Biomedical Engineering, Automation Engineering, ICT
Industrial Engineering, Mechanical Engineering, Metallurgy, Materials, Nanotechnology, Aeronautical, Naval and Aerospace Engineering

This project falls within the classical topic of the Calculus of Variations and its relations with Partial Differential Equations (PDEs). This is essentially Hilbert's 19th and 23rd problems, which concern the regularity of minimizing solutions to nonlinear elliptic PDEs. An essential progress on Hilbert's 19th problem was made in 1957 by Ennio De Giorgi and, independently, by John Forbes Nash. This result allowed attacking several central problems in the Calculus of Variations and in Geometry, a very important one being the regularity of minimal hypersurfaces of R^n. However, closely related questions to the theory of minimal surfaces remain still open, in particular within the theory of Allen-Cahn phase transitions. We will treat them both in the classical and in the fractional settings.

Nonlinear equations with fractional diffusion arise naturally in the presence of stochastic processes with jumps, and have become one of the hottest topics within PDEs. They have applications to American options in Mathematical Finance, propagation of diseases and plagues in the Earth, as well as to Image Processing, Fluid Mechanics, and Geometry. Equations with fractional diffusion are integro-differential equations. Thus, they are nonlocal equations. The most canonical example of such operators is the fractional Laplacian, a nonlocal elliptic operator of order 2s (for instance, the square root of the Laplacian -or half-Laplacian- corresponds to s being 1/2). The PDE group in the UPC has been pioneer in the study of fractional diffusions, starting with the paper [Cabré & Solà-Morales, CPAM, 2005]. The works by the groups of Caffarelli, Figalli, Savin, Silvestre, and Valdinoci have also played a central role in the development of the theory.

Our project will focus its efforts on two concrete topics:

a) The classification of minimizers to the Allen-Cahn equation (which approximate minimal surfaces) is still to be completed, especially in dimension 8. Some results have established (almost totally) a conjecture of Ennio De Giorgi from 1978: [Ambrosio-Cabré, JAMS 2000], [Savin, Ann. of Math. 2009], and [del Pino et al., Ann. Of Math. 2011]. One of its versions is well understood except for n=8.

b) There are analogue open questions for the fractional version of the Allen-Cahn equation, which will be of our interest. In particular, the limiting objects in the fractional Allen-Cahn equation are nonlocal (or fractional) minimal surfaces -a beautiful new geometric theory extending classical minimal surfaces. They were introduced in the seminal paper of [Caffarelli-Roquejoffre-Savin, CPAM 2010] and, while much progress has been done, still many well-known results on minimal surfaces remain open in the nonlocal setting.