SINGULAR INTEGRALS AND RECTIFIABILITY

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Abstract. Rectifiability is a geometric property of measures that turned out to be deeply connected with singular integrals, a class of operators very important in Harmonic Analysis and PDE's. A particularly relevant singular integral is the Riesz transform, which is intimately related with the Laplacian and its fundamental solution.

In 2014 Nazarov, Tolsa and Volberg proved an important characterization of uniform rectifiability, a quantitative version of rectifiability, in terms of the L^2-boundedness of the Riesz transform: if μ is a measure on $\mathbb{R}^{n+1}$ such that $\mu(B(x,r)) \approx r^n$ for every ball centered at its support, then μ is uniformly rectifiable if and only if its associated n-Riesz transform

$$\mathcal{R}_\mu f(x) = \int \frac{x-y}{|x-y|^{n+1}} f(y) d\mu(y)$$

is bounded on $L^2(\mu)$.

In this talk I will make an overview on these aspects. Its final goal is to present the results of a recent joint work with Laura Prat and Xavier Tolsa that extends the one of Nazarov, Tolsa and Volberg to a class of operators that are the analogue of the Riesz transform in the context of elliptic PDE's.

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