“Sobolev flows of non-Lipschitz vector fields”

We show that vector fields with exponentially integrable space derivatives admit a well defined flow of homeomorphisms $X(t,\cdot)$ belonging to the local Sobolev space $W^{1,p(t)}_{loc}$ for some $p(t)>1$, at least for small times. When the field is certain Riesz potential of a bounded function, the result becomes global-in-time, due to deep results from Geometric Function Theory. The local-in-time result also applies to the flows arising from Yudovich solutions to the planar incompressible Euler system. Somehow, our result lies in the midpoint between the classical Cauchy-Lipschitz theory (i.e. Lipschitz vector fields provide bilipschitz flows), and the much more recent examples by Jabin or Alberti-Crippa-Mazzucato (i.e. Sobolev vector fields may give non-Sobolev flows).

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