I BGSMath Junior Meeting

December 11, 2015

09:20 - 09:30    Short Presentation
09:30 - 10:00    Eslam Esaam
10:00 - 10:30    Piermarco Milione
10:30 - 11:00    Guillem Blanco
11:00 - 11:30    Coffee Break
11:30 - 12:00    Carlos Andrés Giraldo
12:00 - 12:30    Amanda Fernández-Fontelo
12:30 - 13:00    Anna Kiesenhofer
13:00 - 14:30    Time for lunch (not included)
14:30 - 15:00    Roger Ten-Valls
15:00 - 15:30    Roland Barrolleta
15:30 - 16:00    Alexandre Sierra
16:00 - 16:30    Coffee Break
16:30 - 17:00    Ariadna Farrés
17:00 - 17:30    Giulia Binotto
17:30 - 18:00    Marc Jorba-Cuscó
On loci of smooth plane non-singular curves with a fixed non-trivial automorphism group

Eslam Esaam Ebrahim Farag, UAB

Let \( M_g \) be the moduli space of smooth, genus \( g \) curves over an algebraically closed field \( K \) of zero characteristic, \( M_g(G) \) be the subset of curves \( \delta \) such that \( G \) is isomorphic to a subgroup of \( \text{Aut}(\delta) \), and let \( \widetilde{M}_g(G) \) be the subset of curves \( \delta \) such that \( G \cong \text{Aut}(\delta) \). Now, for an integer \( d \geq 4 \), let \( M^{Pl}_g \) be the subset of \( M_g \) representing smooth, genus \( g \) plane curves of degree \( d \), and consider the sets \( M^{Pl}_g(G) := M^{Pl}_g \cap M_g(G) \) and \( \widetilde{M}^{Pl}_g(G) := \widetilde{M}_g(G) \cap M^{Pl}_g \).

This talk is devoted to present the results that have been obtained on these loci. For instance, some aspects of the irreducibility of \( \widetilde{M}^{Pl}_g(G) \) and its interrelation with the existence of “normal forms”, the analogy of Henn’s results of plane quartics, but for degree 5 curves (jointly with Francesc Bars), and also an account on the set of twists of such loci (jointly with Francesc Bars and Elisa Lorenzo).

Uniformization of Shimura curves

Piermarco Milione, UB

In this talk we claim to introduce the audience to one of the research interests of the Barcelona Number Theory Group, which is the study of Shimura curves and their uniformizations. We will exhibit several examples of uniformizations of curves, paying special attention on Shimura curves and their \( p \)-adic uniformizations.

Effective computation of base points of two dimensional ideals

Guillem Blanco, UPC

In this work we present an algorithm to compute the weighted cluster of base points given any set of generators of the ideal \( I \). The weighted cluster of base points of an ideal \( I \) determines the equisingular type of generic curves with equations in \( I \). In order to make the algorithm computationally feasible the Puiseux expansion of the generators of the ideal \( I \) is used. To that end, we present a novel modification of the Newton-Puiseux algorithm that can compute the complete Puiseux factorization of any input polynomial, i.e. the Newton-Puiseux algorithm is no longer restricted to reduced inputs. Finally, a completely functional implementation of these algorithms is presented in the Macaulay2 software.
Mineral Fibrations
Carlos Andrés Giraldo Hernández, UAB

By using the structure of simplicial cofibrantly generated model category over the category $S^C$ of $C$-diagrams of simplicial sets (where $C$ is a small category), we formulate one definition of minimality for $C$-diagrams that are free. When $C$ is an EI-category with a finite number of objects, it is possible to show that any free $C$-diagram $X$ has a minimal model with good properties. Using this tool, we are able to classify fibrations in $S^C$ whose base space is a constant diagram. Moreover, when the category $C$ is a rooted tree, it is possible to classify fibrations in $S^C$ whose base space is an arbitrary $C$-diagram. Joint work with Carles Broto and Ramón Flores.

Some new models for discrete data
Amanda Fernández-Fontelo, UAB

On the one hand, one of our lines of research is based on the development of new temporal models for discrete series. We are building extended models of the well-known INAR to deal with real-world examples. That is, we have developed a temporal model which is able to adjust the possible under-reportation of several diseases. We have also built an extension of the INAR($p$) model which allows us to specify Hermite innovations and time dependent parameters. On the other hand, we are also working with the problem of underdispersion in discrete variables. It is less common than overdispersion leading us to develop a general model which can be adapt the problem. In the literature there are few models dealing with underdispersion. Many of them are not general model, but models for specific contexts. Currently, we have detected underdispersion in dosimetry biological data in which existing models propose an empirical solution. Now, we are encouraged to set a model which fits this kind of data.

Action-angle coordinates for $b$-symplectic manifolds
Anna Kiesenhofer, UPC

The Arnold-Liouville theorem describes the symplectic structure around a compact Liouville torus of an integrable system on a symplectic manifold. I will present a similar theorem for $b$-symplectic manifolds, i.e. manifolds where the symplectic form has a specific kind of singularity. I will explain an elegant way to understand this result in terms of a special kind of $b$-cotangent lift of torus actions.

Cyclic Codes over a Family of Local Rings and their Binary Images
Roger Ten-Valls, UAB

Let $p_1, p_2, \ldots, p_t$ be different prime numbers with $t \geq 1$, and let $\Delta = p_1^{k_1} p_2^{k_2} \cdots p_t^{k_t}$. Let $\{u_{p_i,j}\}_{1 \leq j \leq k_i}$ be a set of indeterminants. We construct a family of commutative local rings,

$$R_\Delta = R_{p_1^{k_1} p_2^{k_2} \cdots p_t^{k_t}} = \mathbb{F}_2[u_{p_1,1}, \ldots, u_{p_1,k_1}, u_{p_2,1}, \ldots, u_{p_2,k_2}, \ldots, u_{p_t,k_t}] / \langle u_{p_i,j}^{p_i} = 0 \rangle,$$

where the indeterminants $\{u_{p_i,j}\}_{1 \leq j \leq k_i}$ commute. We study the ideal structure of $R_\Delta$, the algebraic structure of cyclic codes over this family of rings, and we determine the relation between them. We give a canonical Gray map such that we can produce binary quasi-cyclic codes of index $\Delta$ in the Hamming space as images of cyclic codes over $R_\Delta$.

PD-sets for binary Hadamard codes
Roland D. Barrolleta, UAB

A binary Hadamard code of length $n$ is a binary code with $2^n$ codewords and minimum distance $n/2$. A $\mathbb{Z}_4$-linear Hadamard code is a binary Hadamard code obtained as the Gray map image of a linear code over $\mathbb{Z}_4$. Permutation decoding is a technique, introduced in 1964 by MacWilliams for linear codes, which involves finding special subsets, called PD-sets, of the permutation automorphism group of a code in order to assist in decoding. Recently, an alternative permutation decoding method for $\mathbb{Z}_4$-linear codes (not necessarily linear) was presented. However, this method assumes that we know an appropriate PD-set for such codes.

In this talk, we explain how to obtain $s$-PD-sets of size $s + 1$, as sets of invertible matrices satisfying certain conditions, for binary linear and $\mathbb{Z}_4$-linear Hadamard codes and we prove that such sets are of minimum size. Finally, we present two recursive constructions to obtain $s$-PD-sets for these families of codes.

Combinatorial Game Theory
Alexandre Sierra Ballarín, UPC

Combinatorial Game Theory (CGT) is a branch of applied mathematics that studies two-player perfect information games with no random elements. Many of these games decompose in such a way that we can determine the outcome of a game from its components. However this is the case only when the rules include the normal play convention, which says that the first player unable to move is the loser. That is not the case in many classic games, like Chess or Go. Dots-and-Boxes is a well-known game in which players try to claim more boxes than their opponent, in which CGT has shown some success. We also consider the game of Nimstring, which has almost the same rules as Dots-and-Boxes, slightly modified by replacing the winning condition by the normal play convention so we can apply the theory of impartial combinatorial games. Although altering the winning condition leads to a completely different game, both games are strongly linked. We present some results about Dots-and-Boxes and Nimstring.
Dynamics and Control of a Solar Sail in the RTBP

Ariadna Farrés, UB

In this talk we want to describe the natural dynamics of a spacecraft in the Earth-Sun system. We will show how to use the information on the dynamics of the system to: (a) find target orbits for different mission applications and (b) control strategies to remain close to unstable configurations. As a dynamical model we will consider the Restricted Three Body Problem. Where we assume that the Earth and Sun are point masses orbiting around their mutual centre of mass in a circular way. The spacecraft is a mass-less particle that is affected by their gravitational attraction but does not affect them. Moreover, we will include the effect of the solar radiation pressure, to consider the special case of a spacecraft is propelled by a Solar Sail. We will describe the different invariant objects in the system: equilibrium points, periodic and quasi-periodic orbits and their associated stable and unstable manifolds. We will relate all these objects to possible mission applications and give a brief idea on how to derive station keeping strategies to remain close to these objects.

Almost sure convergence to complex Brownian motion

Giulia Binotto, UB

We consider a sequence of processes, depending on a parameter $\theta \in (0, 2\pi)$, constructed from a unique Poisson processes and a family of independent Bernoulli random variables. We show that the sequence converges weakly to a complex Brownian motion and we build realizations of these processes that converge almost surely to a complex Brownian motion. We also derive a rate of convergence. The results are based on a theorem of Skorohod. These results are extended in order to build a family of processes that converges almost surely to a d-dimensional standard Brownian motion for any $d \geq 1$. Using this procedure we can simulate easily independent Brownian motions.

On the wild winding process

Marc Jorba-Cuscó, UB

The quasiperiodic case is more interesting. Not every linear quasiperiodic differential equation is reducible. This has to be taken into account when studying bifurcations of quasiperiodic solutions. The reducible ones are analogues to the periodic case and can be studied looking at the eigenvalues of the reduced matrix. On the other hand, there is a large amount of open questions related to the nonreducible case. We are going to study a very simple model, an affine quasiperiodic skew-product, a dynamical system induced by a map. The system has an nonreducible invariant curve with quasi-periodic dynamics. We study a mechanism of fractalization that takes place when we move the parameters to the bifurcation at zero Lyapunov exponent.